

THE UPPER BOUNDS OF NON-REAL EIGENVALUES FOR INDEFINITE p -LAPLACIAN WITH GENERAL SEPARATED BOUNDARY CONDITIONS

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Abstract. In this paper, the upper bounds of non-real eigenvalues of indefinite p -Laplacian problems with general Sturm-Liouville (S-L) type separated boundary conditions are studied. The upper bounds of imaginary parts and absolute values of non-real eigenvalues are given by using the method of bounded variation.

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