

STRICT SINGULARITY AND CLOSED RANGE OF VOLTERRA INTEGRATION OPERATOR ON DIRICHLET–MORREY TYPE SPACES

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Abstract. The strict singularity and closed range property of the Volterra integral operator T_g with the symbol g on the Dirichlet–Morrey type space $\mathcal{D}_{\lambda,k}$ were investigate in this paper.

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