

SOME INEQUALITIES RELATED TO HARMONICALLY CONVEXITY WITH APPLICATIONS IN TSALLIS ENTROPY AND MEANS

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Abstract. In this paper, we give some new inequalities related to harmonically convex functions. In fact, we introduce a certain function F including a harmonically convex function and using Jensen and Jensen-Mercer inequalities to obtain some new inequalities. Finally, we give some application for these inequalities in information theory, Tsallis entropy, mean theory and approximation theory.

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