

## REFINEMENTS OF ACZÉL–POPOVICIU AND BELLMAN INEQUALITIES FOR LINEAR ISOTONIC FUNCTIONALS

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**Abstract.** A refinement of reverse Jensen inequality for linear isotonic functionals is established. As consequences, refinements of the Aczél–Popoviciu and Bellman inequalities for linear isotonic functionals are obtained. Some particular inequalities are also deduced.

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