

STRONG LAWS FOR NONSTATIONARY GARCH MODELS

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Abstract. This paper studies the basic limit theories for the volatilities in nonstationary GARCH(1,1) models. Those include the classical results such as the Marcinkiewicz-Zygmund strong law of large numbers and the Hartman-Wintner law of the iterated logarithm. The main results precisely characterize asymptotic behaviors of the volatilities in nonstationary GARCH(1,1) models, and provide more insight into this top. Some numerical simulations are provided to verify the validity of theoretical results.

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