

## A HARDY TYPE INEQUALITY ON THE UNIT SPHERE

XING YIN AND SONGTING YIN\*

**Abstract.** We establish another Hardy type inequality on the unit sphere and also obtain the corresponding sharp constant.

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