

MUCKENHOUPT WEIGHTS ASSOCIATED WITH A CLASS OF HOMOGENEOUS TREES

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Abstract. In this paper, the authors introduce the Muckenhoupt weights on a class of homogeneous trees, study some important properties of Muckenhoupt weights, and establish an equivalence of Muckenhoupt weights on the trees. As applications, the characterizations of the maximal operator M associated with admissible trapezoids on weighted Lebesgue spaces are obtained.

Mathematics subject classification (2020): 42B25, 05C05, 42B35.

Keywords and phrases: Homogeneous trees, Lebesgue spaces, exponential growth, Muckenhoupt weight, maximal operator.

REFERENCES

- [1] L. ARDITTI, *Analysis on weighted homogeneous trees*, Master's thesis (2018).
- [2] L. ARDITTI, A. TABACCO AND M. VALLARINO, *Hardy spaces on weighted homogeneous trees*, in *Advances in microlocal and time-frequency analysis*, 21–39, Appl. Numer. Harmon. Anal., Birkhäuser/Springer, Cham.
- [3] L. ARDITTI, A. TABACCO AND M. VALLARINO, *BMO spaces on weighted homogeneous trees*, J. Geom. Anal. **31** (2021), no. 9, 8832–8849.
- [4] A. CAYLEY, *On the theory of analytic forms called trees*, Collected Mathematical Papers of Arthur Cayley, Cambridge Univ. Press, **3** (1890) 242–246.
- [5] P. CARTIER, *Harmonic analysis on trees*, in *Harmonic analysis on homogeneous spaces* (Proc. Sympos. Pure Math., vol. XXVI, Williams Coll., Williamstown, Mass., 1972), pp. 419–424, Proc. Sympos. Pure Math., vol. XXVI, Amer. Math. Soc., Providence, RI.
- [6] R. R. COIFMAN AND C. L. FEFFERMAN, *Weighted norm inequalities for maximal functions and singular integrals*, Studia Math. **51** (1974), 241–250.
- [7] W.-K. CHEN, *Applied graph theory*, second revised edition, North-Holland Series in Applied Mathematics and Mechanics, vol. 13, North-Holland, Amsterdam-New York-Oxford, 1976.
- [8] L. R. FOULDS, *Graph theory applications*, Universitext, Springer, New York, 1992.
- [9] W. HEBISCH AND T. STEGER, *Multipliers and singular integrals on exponential growth groups*, Math. Z. **245** (2003), no. 1, 37–61.
- [10] X. HAO, B. D. LI AND S. YANG, *The Hardy-Littlewood maximal operator on discrete weighted Morrey spaces*, Acta Math. Hungar. **172** (2024), no. 2, 445–469.
- [11] A. KORÁNYI AND M. A. PICARDELLO, *Boundary behaviour of eigenfunctions of the Laplace operator on trees*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **13** (1986), no. 3, 389–399.
- [12] B. MUCKENHOUP, *Weighted norm inequalities for the Hardy maximal function*, Trans. Amer. Math. Soc. **165** (1972), 207–226.
- [13] A. G. SETTI, *L^p and operator norm estimates for the complex time heat operator on homogeneous trees*, Trans. Amer. Math. Soc. **350** (1998), no. 2, 743–768.
- [14] A. V. SOBOLEV AND M. Z. SOLOMYAK, *Schrödinger operators on homogeneous metric trees: spectrum in gaps*, Rev. Math. Phys. **14** (2002), no. 5, 421–467.
- [15] S. H. SAKER et al., *Some basic properties and fundamental relations for discrete Muckenhoupt and Gehring classes*, Adv. Difference Equ. **2021** (2021), no. 8, 1–22.
- [16] S. H. SAKER AND R. P. AGARWAL, *Theory of discrete Muckenhoupt weights and discrete Rubio de Francia extrapolation theorems*, Appl. Anal. Discrete Math. **15** (2021), no. 2, 295–316.

- [17] D. B. WEST, *Introduction to graph theory*, Prentice Hall, Upper Saddle River, NJ, 1996.