

ON NEW LYAPUNOV-TYPE INEQUALITY FOR THE DIRICHLET PROBLEM OF THE FRACTIONAL BAGLEY-TORVIK DIFFERENTIAL EQUATION

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Abstract. This paper discusses a class of fractional Bagley-Torvik differential equations under Dirichlet boundary conditions and establishes a new Lyapunov-type inequality. Firstly, by proving an auxiliary lemma, the discussed boundary value problem is effectively transformed into an integral equation involving the Green's function. Secondly, an upper bound estimate for the Green's function is provided. Finally, using a priori estimation methods, the corresponding Lyapunov-type inequality is derived.

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