

## ON THE GENERALIZED JORDAN–VON NEUMANN TYPE CONSTANT AND THE FIXED POINT PROPERTY FOR MULTIVALUED NONEXPANSIVE MAPPINGS

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**Abstract.** In this paper, we show some sufficient conditions on a Banach space  $X$  concerning the generalized von Neumann-Jordan type constant  $C_{-\infty}^{(p)}(a, X)$ , the coefficient  $R(1, X)$  and the coefficient of weak orthogonality, which imply the existence of fixed points for multivalued nonexpansive mappings.

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