

BERNSTEIN—DOETSCH THEOREM FOR GEOMETRIC CONVEX FUNCTIONS

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Abstract. The celebrated Bernstein–Doetsch Theorem was under consideration of many researchers for last ten decades. In the present paper we give this Theorem in the geometric convexity. The characterization of the geometric convexity is given as well. Finally, we show that the Jensen geometric convex function can be extended to geometric convex.

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