

ADVANCED NUMERICAL RADIUS INEQUALITIES

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Abstract. In this article, we present some new inequalities for the numerical radius of products of Hilbert space operators. In particular, we show that if $S, T \in \mathbb{B}(\mathbb{H})$, then for any $\alpha, \beta > 0$

$$\omega^r(S^*T) \leq \frac{\sqrt{2\alpha\beta}}{2}\omega\left(\frac{(T^*T)^r}{\alpha} + i\frac{(S^*S)^r}{\beta}\right),$$

for any $r \geq 1$. Some consequences that generalize some results from the literature are discussed.

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