

BOUNDEDNESS OF AVERAGING OPERATORS ON NON-DOUBLING MANIFOLDS WITH ENDS

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Abstract. In this paper, we explicitly calculate the best constant for weak-type of the operator S_δ which averages $f \in L^p(\mathbb{R}^n)$ over $B(x, \delta|x|)$, introduced by Christ and Grafakos in Proc. Amer. Math. Soc. 123 (1995) 1687–1693. Let M be a non-doubling manifold with two ends $\mathbb{R}^m \# \mathbb{R}^n$ with $m > n \geq 2$. We also show the weak type of the operator S_δ on $L^p(M)$ and $L^p(M)$ boundedness of the operators S_1 and S_2 .

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