

SINGULAR VALUE INEQUALITIES RELATED TO THE GEOMETRIC MEAN

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Abstract. In this short note, we obtain some singular value inequalities associated with the geometric mean of two positive definite matrices. Our results are refinements of some known inequalities.

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