

## A NEW $p$ -NUMERICAL RADIUS

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**Abstract.** The  $p$ -numerical radius of a Hilbert space operator  $A$  is defined as

$$w_p(A) = \sup_{\theta \in \mathbb{R}} \left\| \operatorname{Re} \left( e^{i\theta} A \right) \right\|_p$$

for  $0 < p \leq \infty$ , where  $\operatorname{Re}(\cdot)$  is the real part and  $\|\cdot\|_p$  is the Schatten  $p$ -norm (quasi-norm). In this paper, a more natural  $p$ -numerical radius is defined as

$$n_p(A) = \sup_{\{\phi_j\} \text{ o.n.s.}} \left( \sum_j |\langle A\phi_j, \phi_j \rangle|^p \right)^{1/p}$$

for  $0 < p \leq \infty$ . Properties and inequalities related to this new  $p$ -numerical radius are given.

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