

HÖLDER TYPE INEQUALITIES FOR MATRICES

TSUYOSHI ANDO AND FUMIO HIAI

Abstract. We discuss Hölder type inequalities involving $(A^p + B^p)^{1/p}$ for positive semi-definite matrices A, B . Matrix or trace inequalities of Hölder type as well as weak majorizations of similar type are obtained. Also we give counter-examples for expected Hölder type inequalities.

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