

AN UPPER BOUND FOR THE ZEROS OF THE CYLINDER FUNCTION $C_\nu(x)$

ÁRPÁD ELBERT AND ANDREA LAFORGIA

Abstract. For large values of ν ($\nu > 0$) the k -th positive zero of the cylinder function $C_\nu(x) = J_\nu(x) \cos \alpha - Y_\nu(x) \sin \alpha$, $0 \leq \alpha < \pi$, has the asymptotic expansion

$$j_{\nu\kappa} = \nu + \gamma_\kappa \nu^{1/3} + \frac{3}{10} \gamma_\kappa^2 \nu^{-1/3} + \mathcal{O}(\nu^{-1})$$

where $\kappa = k - \alpha/\pi$, $\gamma_\kappa = -a_\kappa 2^{-1/3}$ and a_κ is the k -th negative zero of the function $Ai(x) \cos \alpha + Bi(x) \sin \alpha$ and $Ai(x)$, $Bi(x)$ denote the Airy functions of the first and the second kind, respectively [1]. We prove that the sum of the first three terms of the asymptotic expansion gives an upper bound for $j_{\nu\kappa}$, provided $\gamma_\kappa \geq \sqrt[3]{35/4} = 2.0606427 \dots$ or $\kappa \geq \kappa_0 = 1.13019788 \dots = 2 - \alpha_0/\pi$ where α_0 is determined by the equation $\cos \alpha_0 Ai(-\sqrt[3]{35/4}) + \sin \alpha_0 Bi(-\sqrt[3]{35/4}) = 0$. This result covers the cases $j_{\nu 2}, j_{\nu 3}, \dots$ and $y_{\nu 2}, y_{\nu 3}, \dots$, for all $\nu > 0$. The main tool used is the well-known Watson formula for $d j_{\nu\kappa}/d\nu$.

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