

NEW GENERALIZATION OF GAUSS–PÓLYA’S INEQUALITY

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Abstract. We consider inequality of Hölder’s type

$$\frac{\int_a^b w_3(x)g(x)dx}{\int_a^b w_3(x)dx} \leq \prod_{i=1}^2 \left(\frac{\int_a^b w_i(x)g(x)dx}{\int_a^b w_i(x)dx} \right)^{1/p_i}, \quad \frac{1}{p_1} + \frac{1}{p_2} = 1,$$

and give a number of results about functions w_1, w_2, w_3 which satisfy the above-mentioned inequality. Also, in a similar way, we consider an inequality of Minkowski’s type.

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