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UPPER AND LOWER SOLUTIONS METHOD FOR HIGHER ORDER DISCRETE BOUNDARY VALUE PROBLEMS

RAVI P. AGARWAL AND FU-HSIANG WONG

Abstract. We shall offer sufficient conditions on the function $f(i, u_1, \cdots, u_{n-1})$, so that the higher order discrete boundary value problem

$$(BVP)\begin{cases} (E)\ \Delta^n u(i-1) + f(i, u(i), \Delta u(i), \cdots, \Delta^{n-2}u(i)) = 0 \text{ for } i \in [1, T+1] \text{ and } n \geqslant 2, \\ (BC)\begin{cases} \Delta^m u(0) = 0, \quad 0 \leqslant m \leqslant n-3, \\ \alpha\Delta^{n-2}u(0) - \beta\Delta^{n-1}u(0) = 0, \\ \gamma\Delta^{n-2}u(T+1) + \delta\Delta^{n-1}u(T+1) = 0, \end{cases} \end{cases}$$

has at least one solution.

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