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TRACE INEQUALITIES FOR MULTIPLE PRODUCTS OF TWO MATRICES

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Abstract. Trace inequalities for multiple products of powers of two matrices are discussed via the method of log majorization. For instance, the trace inequality $|\mathrm{Tr}(A^{p_1}B^{q_1}A^{p_2}B^{q_2}\cdots A^{p_K}B^{q_K})| \leqslant \mathrm{Tr}(AB)$ is obtained for positive semidefinite matrices A, B and $p_i, q_i \geqslant 0$ with $p_1+\cdots+p_K = q_1+\cdots+q_K = 1$ under some additional condition.

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