Mathematical
Inequalities
& Applications
Volume 4, Number 3 (2001), 343–355

CARLEMAN–KNOPP TYPE INEQUALITIES VIA HARDY INEQUALITIES

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Abstract. Some new Carleman-Knopp type inequalities are proved as "end point" inequalities of modern forms of Hardy's inequalities. Both finite and infinite intervals are considered and both the cases $p \leqslant q$ and $q < p$ are investigated. The obtained results are compared with similar results in the literature and the sharpness of the constants is discussed for the power weight case. Moreover, some reversed Carleman-Knopp inequalities are derived and applied.

Mathematics subject classification (2000): 26D15, 26D07.
Key words and phrases: Inequalities, Hardy's inequality, Carleman's inequality, Knopp's inequality.

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Paper MIA-04-33