

QUASI-VARIATIONAL EQUATION

RABIA NESSAH AND CHENGBIN CHU

Abstract. The main result proved in this article is the following. There is an $\bar{x} \in X$ such that $g(\bar{x}) \in C(\bar{x})$ and $\sup_{y \in C(\bar{x})} \Psi(\bar{x}, y) = \Psi(\bar{x}, g(\bar{x}))$ where $C : X \rightarrow 2^Y$ is a correspondence; (g is a function defined over X in Y) and a function Ψ defined over $X \times Y$ in $\mathbb{R}$; X and Y are different sets. This results generalizes the quasi-variational inequation. According to this result, we show the existence of generalized Berge strong equilibrium for a constrained non cooperative game.

Mathematics subject classification (2000): 58E30, 58E35, 52A40, 52A41, 91A10, 91A13.

Key words and phrases: Correspondence, quasi-variational equation, unit continuous partition, generalized Berge strong equilibrium.

REFERENCES

- [1] K. ARROW AND DEBREU, *Existence of Equilibrium for a Competitive Economy*, *Econometrica* **22** (1954), 265–290.
- [2] J. P. AUBIN, *Initiation à l'Analyse Appliquée*. Masson, Paris, 1994.
- [3] J. P. AUBIN, *Mathematical Methods of Game and Economic Theory*, North-Holland, Amsterdam, 1979.
- [4] J. P. AUBIN, *l'Analyse Non Linéaire et ses Motivations Économiques*. Masson, Paris, 1984.
- [5] J. P. AUBIN AND I. EKLAND, *Applied Nonlinear Analysis*. Wiley Interscience, New York, 1984.
- [6] C. BERGE, *Théorie Générale des Jeux à n — personnes*, Gauthier Villars, Paris, 1957.
- [7] C. BERGE, *Espaces Topologiques, Fonctions multivoque*, Dunod, Paris, 1966.
- [8] G. DEBREU, *A Social Equilibrium Existence Theorem*. *Proc. Nat. Acad. Sci. U.S.A.* **38** (1952).
- [9] K. FAN, *Minimax inequality and application*, in *Inequalities*, III (O. Shisha Ed.) Academic Press, New York, 1972.
- [10] M. LARBANI ET R. NESSAH, *Sur l'équilibre fort selon Berge*. *RAIRO Oper. Res.* **35** (2001), 439–451.
- [11] U. MOSCO, *Implicit Variational problems and Quasi-variational Inequalities*, in *Lecture Notes in Math.*, Vol. **543**, pp 83–156, Springer-Verlag, New York / Berlin, 1976.
- [12] R. NESSAH, M. LARBANI AND F. YALAOUL, *g -Maximum Equality*. Soumis pour publication dans la revue *Journal of Nonlinear and Convex Analysis*.
- [13] M. H. SHIH AND K. K. TAN, *Generalizatiéd quasi-variational inequalities in locallyconvex topological vector space*, *J. Math. Anal. Appl* **108** (1985), 333–343.
- [14] W. SHAFFER AND H. SONNENSHEIN, *Equilibrium in Abstract Economies without Ordered Preferences*. *J. Math. Econom* **2** (1975), 345–348.
- [15] G. TIAN AND J. ZHOU, *Quasi-variational Inequalities with non-compact sets*. *J. Math. Appl.* **160** (1991), 583–595.
- [16] G. TIAN AND J. ZHOU, *Quasi-variational Inequalities without the Convexity Assumption*, *J. Math. Anal. Appl.* **172**, 289–299 (1993).
- [17] J. X. ZHOU AND G. CHEN, *Diagonal Convexity Conditions for Problems in Convex Analysis and Quasi-variational Inequalities*, *J. Math. Appl.* **132** (1988), 213–225.
- [18] E. ZEIDLER, *Nonlinear Functional Analysis and its Applications, III : Variational Methods and Optimization*. Springer-Verlag, New York, 1985.