

## IMPROVEMENTS OF SOME INEQUALITIES FOR MOMENTS OF GUESSING FUNCTION

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(communicated by I. Pinelis)

*Abstract.* Further improvements of some inequalities for moments of guessing function are given.

### 1. Introduction

J. L. Massey in [1] considered the problem of guessing the value of a realization of a discrete random variable  $X$  by asking questions of the form: "Is  $X$  equal to  $x$ ?" until the answer is "Yes".

Let  $G(X)$  denote the number of guesses required by a particular guessing strategy for  $X = x$ .

Massey observed that  $E(G(x))$ , the average number of guesses, is minimized by a guessing strategy that possible values of  $X$  sort in decreasing order of probability.

We begin by giving a formal and generalized statement of the above problem, following E. Arikan [2].

Let  $(X, Y)$  be a pair of random variables with  $X$  taking values in a finite set  $\chi$  of size  $n$ ,  $Y$  taking values in countable set  $\psi$ . Call a function  $G(X)$  of the random variable  $X$  a guessing function for  $X$  if  $G : \chi \rightarrow \{1, \dots, n\}$  is one-to-one. Call a function  $G(X|Y)$  a guessing function for  $X$  given  $Y$  if, for any fixed value  $Y = y$ ,  $G(X|y)$  is a guessing function for  $X$ .  $G(X|Y)$  will be thought of as the number of guesses required to determine  $X$  where the value of  $Y$  is given. E. Arikan, in [2], proved some inequalities on the moments of  $G(X)$  and  $G(X|Y)$ .

To simplify the notation, we assume that the  $x_i$  ( $x_i \in \chi$ ) are numbered such that  $x_k$  is always  $k$ th guess. Then  $p$ -moment of guessing function  $G(x)$  is defined by

$$E(G(X)^p) = \sum_{k=1}^n k^p p_k$$

where  $p_k = \Pr(X = x_k)$ ,  $k = 1, \dots, n$ .

Several inequalities for this moment was obtained by S. S. Dragomir and J. van der Hoek [3] and [4]. S. S. Dragomir, J. Pečarić and J. van der Hoek [5] proved the following improvements of results from [3] and [4]:

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**THEOREM 1.1.** *Let  $X$  be a random variable having the probability distribution  $(p_1, \dots, p_n)$ . Then we have the inequality:*

$$\begin{aligned} & \left| E(G(X)^p) - \frac{1}{n} S_p(n) \right| \\ & \leq \left( \frac{1}{n} \left[ \frac{n^2}{4} \right] \right)^{\frac{1}{2}} (P_M - P_m) \left( S_{2p}(n) - \frac{1}{n} S_p(n)^2 \right)^{\frac{1}{2}} \\ & \leq \frac{1}{n} \left[ \frac{n^2}{4} \right] (n^p - 1) (P_M - P_m) \\ & \leq \frac{n(n^p - 1)}{4} (P_M - P_m), \end{aligned}$$

where  $S_p(n) = \sum_{i=1}^n i^p$  and  $P_M = \max_{i=1, \dots, n} \{p_i\}$ ,  $P_m = \min_{i=1, \dots, n} \{p_i\}$ .

**THEOREM 1.2.** *With assumptions of Theorem 1.1, we have inequality*

$$\begin{aligned} & \left| \binom{p+1}{1} E(G(X)^p) - \binom{p+1}{2} E(G(X)^{p-1}) + \dots \right. \\ & \quad \left. + (-1)^{p+1} \binom{p+1}{p} E(G(X)) + (-1)^{p+2} - n^p \right| \\ & \leq \frac{pn^p(P_M - P_m)}{2\sqrt{2p+1}} \leq \frac{(p+1)n^{p+1}}{4} (P_M - P_m), \end{aligned}$$

where  $p \in \mathbb{N}$ ,  $p \geq 1$ .

In this paper we shall give further improvements of these results.

## 2. Around the Grüss inequality

Note that the above results are obtained as consequence of some inequalities which are in connection to the well known Grüss inequality. For example, in proof of Theorem 1.1 the well known inequality of Biernacki, Pidek and Ryll-Nardzewski (see, for instance, [6, Chapter X]) is used. Generalization of this inequality is given by D. Andrica and C. Badea [7] (for a new proof see paper of S. Izumino and J. Pečarić [8]):

**THEOREM 2.1.** *Let  $a = (a_1, \dots, a_n)$  and  $b = (b_1, \dots, b_n)$  be  $n$ -tuples satisfying:*

$$0 < m_1 < M_1, \quad 0 < m_2 < M_2, \quad m_1 \leq a_i \leq M_1, \quad m_2 \leq b_i \leq M_2 \quad (i = 1, \dots, n).$$

*Then for any  $n$ -weight  $p = (p_1, \dots, p_n)$  and  $\sum_{i=1}^n p_i = 1$  we have*

$$\left| \sum_{i=1}^n p_i a_i b_i - \sum_{i=1}^n p_i a_i \cdot \sum_{i=1}^n p_i b_i \right| \leq (M_1 - m_1) (M_2 - m_2) \max_{J \subset I_n} P(J) (1 - P(J)),$$

where  $P(J) = \sum_{i \in J} p_i$  and  $I_n = \{1, \dots, n\}$ .

In 2002, X. Cheng and J. Sun [9] established the following improvement of Grüss inequality:

**THEOREM 2.2.** *Let  $h, g : [a, b] \rightarrow R$  be two integrable functions such that  $\varphi \leq g(x) \leq \Phi$  for some constants  $\varphi, \Phi$  for all  $x \in [a, b]$ , then*

$$\left| \frac{1}{b-a} \int_a^b h(x) g(x) dx - \frac{1}{(b-a)^2} \int_a^b h(x) dx \int_a^b g(x) dx \right| \\ \leq \frac{\Phi - \varphi}{2} \cdot \frac{1}{b-a} \int_a^b \left| h(x) - \frac{1}{b-a} \int_a^b h(y) dy \right| dx.$$

In 2002, P. Cerone and S. S. Dragomir [10] gave the weighted version of Theorem 2.2 as well as the following discrete analogue:

**THEOREM 2.3.** *Let  $(a_1, \dots, a_n), (b_1, \dots, b_n)$  be  $n$ -tuples of real numbers with. If  $b \leq b_i \leq B, i \in \{1, \dots, n\}$ , then for any  $n$ -weight  $p = (p_1, \dots, p_n)$  and  $\sum_{i=1}^n p_i = 1$  one has the inequality*

$$\left| \sum_{i=1}^n p_i a_i b_i - \sum_{i=1}^n p_i a_i \cdot \sum_{i=1}^n p_i b_i \right| \leq \frac{(B-b)}{2} \cdot \sum_{i=1}^n p_i \left| a_i - \sum_{i=1}^n p_i a_i \right|.$$

In fact, Theorem 2.3 is an improvement of Theorem 2.1, that is the following results is valid:

**THEOREM 2.4.** *Let  $a = (a_1, \dots, a_n)$  and  $b = (b_1, \dots, b_n)$  be  $n$ -tuples satisfying:*

$$0 < m_1 < M_1, 0 < m_2 < M_2, m_1 \leq a_i \leq M_1, m_2 \leq b_i \leq M_2 \quad (i = 1, \dots, n)$$

*Then for any  $n$ -weight  $p = (p_1, \dots, p_n)$  and  $\sum_{i=1}^n p_i = 1$  we have*

$$\left| \sum_{i=1}^n p_i a_i b_i - \sum_{i=1}^n p_i a_i \cdot \sum_{i=1}^n p_i b_i \right| \leq \frac{(M_2 - m_2)}{2} \cdot \sum_{i=1}^n p_i \left| a_i - \sum_{i=1}^n p_i a_i \right| \\ \leq \frac{(M_2 - m_2)}{2} \cdot \sum_{i=1}^n p_i \sum_{j=1}^n p_j |a_i - a_j| \\ \leq (M_1 - m_1) (M_2 - m_2) \max_{J \subset I_n} P(J) (1 - P(J))$$

*Proof.* We shall use the following result [8, Corollary 2.4]:

$$\sum_{(i,j) \in \Delta} p_i p_j |a_i - a_j| \leq (M_1 - m_1) \max_{J \subset I_n} P(J) (1 - P(J))$$

where  $\Delta = \{(i, j) \in I_n \times I_n; i < j\}$ . Then we have

$$\begin{aligned} & \frac{(M_2 - m_2)}{2} \cdot \sum_{i=1}^n p_i \left| a_i - \sum_{i=1}^n p_i a_i \right| = \frac{(M_2 - m_2)}{2} \cdot \sum_{i=1}^n p_i \left| \sum_{j=1}^n p_j a_i - \sum_{j=1}^n p_j a_j \right| \\ & = \frac{(M_2 - m_2)}{2} \cdot \sum_{i=1}^n p_i \left| \sum_{j=1}^n p_j (a_i - a_j) \right| \leq \frac{(M_2 - m_2)}{2} \cdot \sum_{i=1}^n p_i \sum_{j=1}^n p_j |a_i - a_j| \\ & \leq (M_2 - m_2) \sum_{(i,j) \in \Delta} p_i p_j |a_i - a_j| \leq (M_1 - m_1) (M_2 - m_2) \max_{J \subset I_n} P(J) (1 - P(J)) \end{aligned}$$

□

We shall use un-weighted version of Theorem 2.3, that is the following result:

**THEOREM 2.3'.** *Let  $(a_1, \dots, a_n)$ ,  $(b_1, \dots, b_n)$  be  $n$ -tuples of real numbers with. If  $b \leq b_i \leq B$ ,  $i \in \{1, \dots, n\}$  then one has the inequality*

$$\left| \frac{1}{n} \sum_{i=1}^n a_i b_i - \frac{1}{n} \sum_{i=1}^n a_i \cdot \frac{1}{n} \sum_{i=1}^n b_i \right| \leq (B - b) \cdot \frac{1}{n} \cdot \sum_{a_i > \frac{1}{n} \sum_{i=1}^n a_i} \left( a_i - \frac{1}{n} \sum_{i=1}^n a_i \right)$$

*Proof.* If we put  $p_i = \frac{1}{n}$ , ( $i = 1, \dots, n$ ) in Theorem 2.3 we have the inequality:

$$\left| \frac{1}{n} \sum_{i=1}^n a_i b_i - \frac{1}{n} \sum_{i=1}^n a_i \cdot \frac{1}{n} \sum_{i=1}^n b_i \right| \leq \frac{(B - b)}{2} \cdot \frac{1}{n} \cdot \sum_{i=1}^n \left| a_i - \frac{1}{n} \sum_{i=1}^n a_i \right|$$

On the other-hand side we have

$$\sum_{i=1}^n \left( a_i - \frac{1}{n} \sum_{i=1}^n a_i \right) = \sum_{a_i > \frac{1}{n} \sum_{i=1}^n a_i} \left( a_i - \frac{1}{n} \sum_{i=1}^n a_i \right) + \sum_{a_i \leq \frac{1}{n} \sum_{i=1}^n a_i} \left( a_i - \frac{1}{n} \sum_{i=1}^n a_i \right) = 0,$$

and

$$\sum_{i=1}^n \left| a_i - \frac{1}{n} \sum_{i=1}^n a_i \right| = 2 \cdot \sum_{a_i > \frac{1}{n} \sum_{i=1}^n a_i} \left( a_i - \frac{1}{n} \sum_{i=1}^n a_i \right).$$

Hence we obtain our result. □

We shall also use the following:

COROLLARY 2.1. Let  $g, h : [a, b] \rightarrow R$  be two integrable functions such that  $\varphi \leq g(x) \leq \Phi$  for some constants  $\varphi, \Phi$  for all  $x \in [a, b]$  and  $h(x) = (x - a)^p$ ,  $p > 0$ , than

$$\left| \frac{1}{b-a} \int_a^b h(x) g(x) dx - \frac{1}{(b-a)^2} \int_a^b h(x) dx \int_a^b g(x) dx \right| \leq (\Phi - \varphi) \cdot \frac{p}{(p+1)^2 (p+1)^{\frac{1}{p}}} (b-a)^p.$$

*Proof.* Using Theorem 2.2 we can state:

$$\left| \frac{1}{b-a} \int_a^b h(x) g(x) dx - \frac{1}{(b-a)^2} \int_a^b h(x) dx \int_a^b g(x) dx \right| \leq \frac{\Phi - \varphi}{2} \cdot \frac{1}{b-a} \int_a^b \left| h(x) - \frac{1}{b-a} \int_a^b h(y) dy \right| dx.$$

□

Now, Corollary 2.1 is a simple consequence of the following result:

LEMMA 2.1. Let  $h : [a, b] \rightarrow R$ ,  $h(x) = (x - a)^p$  where  $p > 0$  than

$$\begin{aligned} \frac{1}{b-a} \int_a^b \left| h(x) - \frac{1}{b-a} \int_a^b h(y) dy \right| dx &= \frac{1}{b-a} \int_a^b \left| (x-a)^p - \frac{1}{b-a} \int_a^b (y-a)^p dy \right| dx \\ &= \frac{2p}{(p+1)^2 (p+1)^{\frac{1}{p}}} \cdot (b-a)^p. \end{aligned}$$

*Proof.* Obviously, we have

$$\begin{aligned} \frac{1}{b-a} \int_a^b \left| h(x) - \frac{1}{b-a} \int_a^b h(y) dy \right| dx &= \frac{1}{b-a} \int_a^b \left| (x-a)^p - \frac{1}{b-a} \int_a^b (y-a)^p dy \right| dx \\ &= \frac{1}{b-a} \int_a^b \left| (x-a)^p - \frac{(b-a)^p}{p+1} \right| dx. \end{aligned}$$

We have

$$\left| (x-a)^p - \frac{(b-a)^p}{p+1} \right| = (x-a)^p - \frac{(b-a)^p}{p+1} \quad \text{for } (x-a)^p \geq \frac{(b-a)^p}{p+1}$$

or  $x_0 = a + \frac{b-a}{(p+1)^{\frac{1}{p}}} \leq x \leq b$

$$\left| (x-a)^p - \frac{(b-a)^p}{p+1} \right| = \frac{(b-a)^p}{p+1} - (x-a)^p \quad \text{for } (x-a)^p \leq \frac{(b-a)^p}{p+1}$$

$$\text{or } a \leq x \leq a + \frac{b-a}{(p+1)^{\frac{1}{p}}} = x_0,$$

and hence we obtained

$$\begin{aligned} \frac{1}{b-a} \int_a^b \left| h(x) - \frac{1}{b-a} \int_a^b h(y) dy \right| dx &= \frac{1}{b-a} \int_a^b \left| (x-a)^p - \frac{(b-a)^p}{p+1} \right| dx \\ &= \frac{1}{b-a} \int_a^{x_0} \left( \frac{(b-a)^p}{p+1} - (x-a)^p \right) dx + \int_{x_0}^b \left( (x-a)^p - \frac{(b-a)^p}{p+1} \right) dx \\ &= \frac{1}{b-a} \left( \frac{(b-a)^p}{p+1} (x_0-a) - \frac{(x_0-a)^{p+1}}{p+1} + \frac{(b-a)^{p+1}}{p+1} \right. \\ &\quad \left. - \frac{(x_0-a)^{p+1}}{p+1} - \frac{(b-a)^p}{p+1} (b-a) \right) \\ &= \frac{1}{(p+1)(b-a)} \left( (b-a)^p (x_0-a-b+a-b+x_0) - 2(x_0-a)^{p+1} \right) \\ &= \frac{2}{(p+1)(b-a)} \left( (b-a)^p (x_0-b) - (x_0-a)^{p+1} \right) \\ &= \frac{2p}{(p+1)^2 (p+1)^{\frac{1}{p}}} (b-a)^p. \end{aligned}$$

### 3. Main results

Now we shall give improvements of Theorem 1.1 and 1.2.

**THEOREM 3.1.** *Let  $X$  be a random variable having the probability distribution  $(p_1, \dots, p_n)$  and  $p > 0$ . Then we have the inequality:*

$$\left| E(G(X)^p) - \frac{1}{n} S_p(n) \right| \leq (P_M - P_m) \cdot c \cdot \left( \frac{1}{n} \cdot S_p(n) - \frac{1}{c} \cdot S_p(c) \right),$$

where

$$S_p(k) = \sum_{i=1}^k i^p, \quad c = \left[ \left( \frac{S_p(n)}{n} \right)^{\frac{1}{p}} \right]$$

$P_M = \max_{i=1, \dots, n} \{p_i\}$ ,  $P_m = \min_{i=1, \dots, n} \{p_i\}$  and  $[x]$  denotes the integer part of  $x$ ,  $x \in \mathbf{R}$ .

*Proof.* Using Theorem 2.3', we can state

$$\left| \frac{1}{n} \sum_{i=1}^n a_i b_i - \frac{1}{n} \sum_{i=1}^n a_i \cdot \frac{1}{n} \sum_{i=1}^n b_i \right| \leq (B - b) \cdot \frac{1}{n} \cdot \sum_{a_i > \frac{1}{n} \sum_{i=1}^n a_i} \left( a_i - \frac{1}{n} \sum_{i=1}^n a_i \right).$$

Now chose  $a_i = i^p$  and  $b_i = p_i$  ( $i = 1, \dots, n$ ) we obtain

$$\left| \sum_{i=1}^n i^p p_i - \frac{1}{n} \sum_{i=1}^n i^p \cdot \sum_{i=1}^n p_i \right| \leq (P_M - P_m) \sum_{i^p > \frac{1}{n} \sum_{i=1}^n i^p} \left( i^p - \frac{1}{n} \sum_{i=1}^n i^p \right).$$

We observe that

$$i^p > \frac{1}{n} \sum_{i=1}^n i^p$$

$$i > \left[ \left( \frac{1}{n} \sum_{i=1}^n i^p \right)^{\frac{1}{p}} \right].$$

Using above results we have:

$$\left| \sum_{i=1}^n i^p p_i - \frac{1}{n} \sum_{i=1}^n i^p \cdot \sum_{i=1}^n p_i \right| \leq (P_M - P_m) \sum_{\left[ \left( \frac{1}{n} \sum_{i=1}^n i^p \right)^{\frac{1}{p}} \right] < i \leq n} \left( i^p - \frac{1}{n} \sum_{i=1}^n i^p \right).$$

Together with:

$$\sum_{\left[ \left( \frac{1}{n} \sum_{i=1}^n i^p \right)^{\frac{1}{p}} \right] < i \leq n} \left( i^p - \frac{1}{n} \sum_{i=1}^n i^p \right) = \sum_{1 \leq i < \left[ \left( \frac{1}{n} \sum_{i=1}^n i^p \right)^{\frac{1}{p}} \right]} \left( \frac{1}{n} \sum_{i=1}^n i^p - i^p \right).$$

We have now:

$$\sum_{\left[ \left( \frac{1}{n} \sum_{i=1}^n i^p \right)^{\frac{1}{p}} \right] < i \leq n} \left( i^p - \frac{1}{n} \sum_{i=1}^n i^p \right) = \sum_{1 \leq i \leq c} \left( \frac{1}{n} S_p(n) - i^p \right)$$

$$= c \cdot \frac{1}{n} S_p(n) - S_p(c) = c \cdot \left( \frac{1}{n} S_p(n) - \frac{1}{c} S_p(c) \right),$$

and the inequality in Theorem 3.1. is obtained.  $\square$

REMARK 3.1. If we choose  $p = 1$ , we get:

$$\left| E(G(X)) - \frac{n+1}{2} \right| \leq \frac{(P_M - P_m)}{2} \left[ \frac{n+1}{2} \right] \left( n - \left[ \frac{n+1}{2} \right] \right)$$

If we choose  $p = 2$ , we get:

$$\begin{aligned} & \left| E(G(X)^2) - \frac{(n+1)(2n+1)}{6} \right| \\ & \leq \frac{(P_M - P_m)}{6} \left[ \left( \frac{n(n+1)(2n+1)}{6} \right)^{\frac{1}{2}} \right] \\ & \quad \cdot \left( (n+1)(2n+1) - \left( \left[ \left( \frac{n(n+1)(2n+1)}{6} \right)^{\frac{1}{2}} \right] + 1 \right) \right) \\ & \quad \cdot \left( 2 \left[ \left( \frac{n(n+1)(2n+1)}{6} \right)^{\frac{1}{2}} \right] + 1 \right) \end{aligned}$$

If we choose  $p = 3$ , we get:

$$\begin{aligned} & \left| E(G(X)^3) - \frac{n(n+1)^2}{4} \right| \leq \frac{(P_M - P_m)}{4} \left[ \left( \frac{n^2(n+1)^2}{4} \right)^{\frac{1}{3}} \right] \\ & \quad \cdot \left( n(n+1)^2 - \left[ \left( \frac{n^2(n+1)^2}{4} \right)^{\frac{1}{3}} \right] \left( \left[ \left( \frac{n^2(n+1)^2}{4} \right)^{\frac{1}{3}} \right] + 1 \right)^2 \right). \end{aligned}$$

COROLLARY 3.1. If we assume that for a given  $\varepsilon > 0$  and natural number  $n$ , we have

$$P_M - P_m \leq \varepsilon / \left\{ c \left( \frac{1}{n} \cdot S_p(n) - \frac{1}{c} \cdot S_p(c) \right) \right\},$$

then

$$\left| E(G(X)^p) - \frac{1}{n} S_p(n) \right| \leq \varepsilon.$$

THEOREM 3.2. With the assumptions of Theorem 3.1, we have the inequality:

$$\begin{aligned} & \left| \binom{p+1}{1} E(G(X)^p) - \binom{p+1}{2} E(G(X)^{p-1}) + \dots \right. \\ & \quad \left. + (-1)^{p+1} \binom{p+1}{p} E(G(X)) + (-1)^{p+2} n^p \right| \\ & \leq (P_M - P_m) \cdot \frac{p}{(p+1)(p+1)^{\frac{1}{p}}} n^{p+1}. \end{aligned}$$

*Proof.* From Corollary 2.1 we have:

$$\left| \frac{1}{b-a} \int_a^b (x-a)^p g(x) dx - \frac{1}{(b-a)^2} \int_a^b (x-a)^p dx \int_a^b g(x) dx \right| \\ \leq (\Phi - \varphi) \cdot \frac{P}{(p+1)^2 (p+1)^{\frac{1}{p}}} (b-a)^p.$$

If we chose  $a = 0$ ,  $b = n$ ,  $g(x) = p_{i+1}$ ,  $x \in [i, i+1)$ ,  $i = 0, \dots, n-1$  and  $\Phi = P_M$ ,  $\varphi = P_m$  we have:

$$\left| \frac{1}{n(p+1)} \left( \binom{p+1}{1} E(G(X)^p) - \binom{p+1}{2} E(G(X)^{p-1}) + \dots \right. \right. \\ \left. \left. + (-1)^{p+1} \binom{p+1}{p} E(G(X)) + (-1)^{p+2} \right) - \frac{1}{n^2} \cdot \frac{n^{p+1}}{p+1} \right| \\ \leq (P_M - P_m) \cdot \frac{P}{(p+1)^2 (p+1)^{\frac{1}{p}}} n^p$$

wherefrom we get our result.  $\square$

REMARK 3.2. If we choose  $p = 1$ , we get:

$$\left| E(G(X)) - \frac{n+1}{2} \right| \leq \frac{1}{8} \cdot n^2 \cdot (P_M - P_m)$$

If we choose  $p = 2$ , we get:

$$\left| E(G(X)^2) - E(G(X)) - \frac{n^2-1}{3} \right| \leq \frac{2\sqrt{3}}{27} \cdot n^3 \cdot (P_M - P_m)$$

COROLLARY 3.2. If we assume that for a given  $\varepsilon > 0$  and natural number  $n$ , we have

$$P_M - P_m \leq \varepsilon (p+1)^2 (p+1)^{\frac{1}{p}} / (pn^{p+1}),$$

then

$$\left| \frac{1}{n(p+1)} \left( \binom{p+1}{1} E(G(X)^p) - \binom{p+1}{2} E(G(X)^{p-1}) + \dots \right. \right. \\ \left. \left. + (-1)^{p+1} \binom{p+1}{p} E(G(X)) + (-1)^{p+2} \right) - \frac{1}{n^2} \cdot \frac{n^{p+1}}{p+1} \right| \leq \varepsilon.$$

REMARK 3.3. The above results are improvements of related results from [5].

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