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ERGODICITY COEFFICIENT AND PERTURBATION BOUNDS FOR CONTINUOUS–TIME MARKOV CHAINS

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Abstract. For the distribution of a finite, homogeneous, continuous-time Markov chain, we derive perturbation bounds in terms of the ergodicity coefficient of the transition probability matrix. Our perturbation bounds improve upon the known results. We give sensitivity bounds for the coefficient of ergodicity, providing a sufficient condition for the uniqueness of the stationary distribution of the perturbed Markov chain. These results are used to obtain estimates of the speed of convergence for singularly perturbed Markov chains.

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