

ON THE LOG-CONVEXITY OF TWO-PARAMETER HOMOGENEOUS FUNCTIONS

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Abstract. Suppose $f(x, y)$ is a positive homogeneous function defined on $\mathbb{U}(\subseteq \mathbb{R}_+ \times \mathbb{R}_+)$, then call $\left(\frac{f(a^p, b^p)}{f(a^q, b^q)}\right)^{\frac{1}{p-q}}$ two-parameter homogeneous function and denote by $\mathcal{H}_f(a, b; p, q)$. If $f(x, y)$ is third differentiable, then the log-convexity with respect to parameters p and q of $\mathcal{H}_f(p, q)$ depend on the sign of $J = (x - y)(xI)_x$, where $I = (\ln f)_{xy}$. As applications a group of chains of inequalities for homogeneous means are established, which generalize, strengthen and unify Tong-po Ling's and Stolarsky's inequalities, and a reversed chain of inequalities for exponential mean (identic mean) is derived, which contains a reversed Stolarsky's inequality. Several estimations of lower and upper bounds of extended mean are presented.

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REFERENCES

- [1] B. C. CARLSON, *The logarithmic mean*, Amer. Math. Monthly, **79**, (1972), 615–618.
- [2] P. CZINDER, Z. PÁLES, *An extension of the Hermite-Hadamard inequality and an application for Gini and stolarsky means*, Journal of Inequalities in Pure and Applied Mathematics, **5**, (2) (2004) [ONLINE] Available online at http://jipam.vu.edu.au/images/167_03_JIPAM/167_03.pdf.
- [3] C. GINI, *Diuna formula comprensiva delle media*, Metron, **13**, (1938), 3–22.
- [4] P. A. HÄSTÖ, *A monotonicity property of ratios of symmetric homogeneous means*, Journal of Inequalities in Pure and Applied Mathematics, **3**, (5) (2002) [ONLINE] Available online at http://jipam-old.vu.edu.au/v3n5/013_02.pdf.
- [5] GAO JIA, JINDE CAO, *A new upper bound of the logarithmic mean*, Journal of Inequalities in Pure and Applied Mathematics, **4**, (4) (2003) [ONLINE] Available online at http://jipam.vu.edu.au/images/088_03/088_03_www.pdf.
- [6] E. B. LEACH, M. C. SHOLANDER, *Extended mean values*, Amer. Math. Monthly, **85**, (1978), 84–90.
- [7] E. B. LEACH, M. C. SHOLANDER, *Extended mean values II*, J. Math. Anal. Appl., **92**, (1983), 207–223.
- [8] T.-P. LIN, *The power mean and the logarithmic mean*, Amer. Math. Monthly, **81**, (1974), 879–883.
- [9] Z. PÁLES, *Inequalities for differences of powers*, J. Math. Anal. Appl., **131**, (1988), 271–281.
- [10] F. QI, *Logarithmically convexities of the Extended Mean values*, Proc. Am. Math. Soc. **130**, (6) (2002), 1787–1796.
- [11] F. QI, *Generalized weighted mean values with two parameters*, Proceedings of the Royal Society of London, Series, Mathematical, Physical and Engineering Sciences, **454**, (1978) (1998), 2723–2732.
- [12] F. QI, *On a two-parameter family of nonhomogeneous mean values*, Tamkang Journal of Mathematics, **29**, (2) (1998), 155–163.
- [13] F. QI, *Generalized Abstracted Mean Values*, Journal of Inequalities in Pure and Applied Mathematics, **1**, (1) (1999) [ONLINE] Available online at http://jipam-old.vu.edu.au/v1n1/013_99.pdf.
- [14] K. B. STOLARSKY, *Generalizations of the Logarithmic Mean*, Math. Mag., **48**, (1975), 87–92.
- [15] K. B. STOLARSKY, *The power and generalized Logarithmic Means*, Amer. Math. Monthly., **87**, (1980), 545–548.

- [16] MING-BAO SUN, *Inequality for two-parameter mean of convex function*, Mathematics in Practice and Theory, **27**, (3) (1997), 193–197.
- [17] ZHEN-HANG YANG, *Exponential mean and logarithmic Mean*, Mathematics in Practice and Theory, **4**, (1987), 76–78.
- [18] ZHEN-HANG YANG, *Inequalities for power mean of convex function*, Mathematics in Practice and Theory, **20**, (1) (1990), 93–96.
- [19] ZHEN-HANG YANG, *Inequalities for general mean of convex function*, Mathematics in Practice and Theory, **33**, (8) (2003), 136–141.
- [20] ZHEN-HANG YANG, *Simple discriminance for convexity of homogeneous functions and applications*, Study in College Mathematics, **7**, (4) (2004), 14–19.