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REMARKS ON A PYTHAGOREAN APPROACH IN BANACH SPACES

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Abstract. We give relations between the parameters $E(X)$ and $f(X)$, introduced by the first author in [4], and other geometric constants. Main results in [4] and [5] are derived and strengthened. More precisely, a wider class of Banach spaces with uniform normal structure is obtained. A relation between $E(X)$ and $E(X^*)$ is also given.

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