

ON THE BOUNDS FOR THE NORMALIZED JENSEN FUNCTIONAL AND JENSEN–STEFFENSEN INEQUALITY

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Abstract. We consider the inequalities of type

$$M \mathcal{J}_n(f, \mathbf{x}, \mathbf{q}) \geq \mathcal{J}_n(f, \mathbf{x}, \mathbf{p}) \geq m \mathcal{J}_n(f, \mathbf{x}, \mathbf{q}),$$

where f is a convex function and $\mathcal{J}_n(f, \mathbf{x}, \mathbf{p}) = \sum_{i=1}^n p_i f(x_i) - f(\sum_{i=1}^n p_i x_i)$, recently introduced by S.S. Dragomir. We give an alternative proof of such inequalities and prove another similar result for the case when f is a convex function on an interval in the real line, while \mathbf{p} and \mathbf{q} satisfy the conditions for Jensen-Steffensen inequality. We show that our result improves the result of Dragomir in this special case. We also prove the integral versions of all our results, including those related to Boas' generalization of Jensen-Steffensen integral inequality.

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