

TOPOLOGICAL AND GEOMETRICAL STRUCTURE OF CALDERÓN-LOZANOVSKIĬ CONSTRUCTION

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Abstract. We study the structure of general Calderón-Lozanovskiĭ construction. First the problem of order continuity of these spaces is studied. Moreover, we find characterization of strict monotonicity and we try to explain why the obtained criteria are not easy to verify in particular cases.

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