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MODULAR INEQUALITIES FOR THE HARDY–LITTLEWOOD AVERAGES

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Abstract. In this paper we establish general inequalities of the Hardy-Littlewood averages. We apply our results to obtained the higher-dimensional form of a strengthened Hardy-Knopp-type inequality. Furthermore, we discuss the inequalities given by Čižmešija et. al. [1], Cochran and Lee [2], Heinig [5], S. Kaijser et. al. [6], Levinson [8], Love [9 10], and Xiao [12], and show that these results are special cases of our results in this paper.

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