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ON THE COMPLETE MONOTONICITY OF QUOTIENT OF GAMMA FUNCTIONS

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Abstract. In this paper, we concern with a conjecture involving the ratio of two gamma functions posed by Qi, Guo and Chen [Math. Inequal. Appl. 9(3)(2006), 427-436]. We also generalize a result of Qi [Theorem 2, J. Comput. Appl. Math. 214 (2008), 610-616].

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