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ESTIMATES FOR LOWER BOUNDS OF EIGENVALUES OF THE KLEIN–GORDON OPERATOR

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Abstract. In this paper, we establish some inequalities for eigenvalues of the Klein-Gordon operator on a bounded domain in an n-dimensional Euclidean space. These inequalities give some sharper estimates for lower bounds of the sums of its first k eigenvalues, which improve the recent results of Yildirim Yolcu (Proc. Amer. Math. Soc. **138** (2010), 4059–4066).

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