

HIGHER ORDER DYNAMIC INEQUALITIES ON TIME SCALES

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Abstract. In this paper, we will prove some new dynamic inequalities of higher orders on time scales. The results contain some continuous and discrete inequalities as special cases. We will prove the results by making use of Hölder’s inequality and Taylor monomials on time scales.

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