

HILBERT-TYPE INEQUALITIES INVOLVING DIFFERENTIAL OPERATORS, THE BEST CONSTANTS, AND APPLICATIONS

VANDANJAV ADIYASUREN, TSERENDORJ BATBOLD AND MARIO KRNIĆ

Abstract. Motivated by some recent results, in this article we derive several Hilbert-type inequalities with a differential operator, regarding a general homogeneous kernel. Moreover, we show that the constants appearing on the right-hand sides of these inequalities are the best possible. The general results are then applied to some particular examples of homogeneous kernels and compared with previously known from the literature.

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