

SOME CHARACTERIZATIONS OF INNER PRODUCT SPACES BY INEQUALITIES

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Abstract. In this paper we present some new characterizations of inner product spaces by using inequalities. First, we explore a classical idea consisting in the transformation of the parallelogram law into an inequality. Then we give some characterizations by using convex and Wright-convex functions.

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