

## RELATIONS BETWEEN TWO CLASSES OF REAL FUNCTIONS AND APPLICATIONS TO BOUNDEDNESS AND COMPACTNESS OF OPERATORS BETWEEN ANALYTIC FUNCTION SPACES

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**Abstract.** Some relations between two classes of real functions, the class of positive lower type  $p$ , denoted by  $\mathfrak{L}_p$ , and the class of positive upper type  $q$ , denoted by  $\mathfrak{U}^q$ , which naturally appear in the definition of some Hardy-Orlicz and Bergman-Orlicz type spaces, are given. Some applications in characterizing the boundedness and compactness of an integral-type operator introduced by S. Stević, from Hardy-Orlicz and Bergman-Orlicz type spaces to a weighted-type space are given.

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