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INEQUALITIES AND BOUNDS FOR A CERTAIN BIVARIATE ELLIPTIC MEAN

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Abstract. This paper deals with a new mean introduced recently by this author. This mean is a degenerate case of the completely symmetric elliptic integral of the second kind. In particular inequalities involving mean under discussion are obtained. Also, bounds in the mean in question are obtained. Bounding expressions are convex combinations of some quantities depending on variables of the mean.

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