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SHARP BOUNDS FOR m–LINEAR HILBERT–TYPE OPERATORS ON THE WEIGHTED MORREY SPACES

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Abstract. On the product of m weighted Morrey spaces, some m-linear operators are shown to be bounded. The operator norm is calculated explicitly. It may be interesting to compare the results for the Hardy operator and the ones for the Hardy-Littlewood maximal operator. In the end of this article, some concrete examples are presented.

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