

## ON THE TRIEBEL-LIZORKIN SPACE BOUNDEDNESS OF MARCINKIEWICZ INTEGRALS ALONG COMPOUND SURFACES

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**Abstract.** In this paper the author present the boundedness of Marcinkiewicz integral operators associated to compound surfaces with rough kernels given by  $h \in \Delta_\gamma(\mathbb{R}_+)$  and  $\Omega \in L(\log^+ L)^{1/2}(S^{n-1}) \cup (\cup_{1 < r < \infty} B_r^{0,-1/2}(S^{n-1}))$  on Triebel-Lizorkin spaces and Besov spaces. The main results of this paper represent improvements as well as natural extensions of many previously known results.

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