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SOME NEW TWO–SIDED INEQUALITIES CONCERNING THE FOURIER TRANSFORM

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Abstract. The classical Hausdorff-Young and Hardy-Littlewood-Stein inequalities do not hold for $p > 2$. In this paper we prove that if we restrict to net spaces we can even derive a two-sided estimate for all $p > 1$. In particular, this result generalizes a recent result by Liflyand E. and Tikhonov S. [7] (MR 2464253)

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