

## WEIGHTED END-POINT WEAK TYPE $(p, p)$ ESTIMATES FOR $g_\lambda^*$ -FUNCTION WITH KERNELS OF LOWER REGULARITIES

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**Abstract.** In 1970, if  $1 < p < 2$  and  $\lambda = 2/p$ , C. Fefferman obtained the end-point weak  $(p, p)$  boundedness of  $g_\lambda^*$ -function. In this paper, the authors essentially improved the result given by C. Fefferman, by showing that the weighted end-point weak type  $(p, p)$  boundedness of  $g_\lambda^*$ -function still holds with lower regularities assumed on the kernel for  $1 < p < 2$  and  $\lambda = 2/p$ . Moreover, similar results can also be extended to parametric Littlewood-Paley  $g_\lambda^*$ -function with more rough kernels.

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