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HIGHER–ORDER QUASIMONOTONICITY AND INTEGRAL INEQUALITIES

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Abstract. The classical Hermite–Hadamard inequality is not merely a consequence of convexity, but also characterizes convexity. Such inequalities hold in the case of higher-order monotonicity in sense of Hopf and Popoviciu with the same characteristic feature. The aim of this note is to extend these results, when the underlying monotonicity is induced by so-called quasipolynomial Chebyshev systems.

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