

FACTORIZATION OF LIPSCHITZ OPERATORS ON BANACH FUNCTION SPACES

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Abstract. Let (X, d) be a pointed metric space. Let $T: X \rightarrow Y_1(\mu)$ and $S: X \rightarrow Y_2(\mu)$ be two Lipschitz operators into two Banach function spaces Y_1 and Y_2 over the same finite measure μ . We show which are the vector norm inequalities that characterize those T and S for which $T = M_g \circ S$, for some multiplication operator $M_g: Y_2 \rightarrow Y_1$. Our ideas give rise to Maurey-Rosenthal type factorization results for Lipschitz operators. We provide some applications on the Lipschitz structure of metric subsets of Banach function spaces.

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