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THE EXACT CONSTANT FOR THE $\ell_1 - \ell_2$ NORM INEQUALITY

SARA BOTELHO-ANDRADE, PETER G. CASAZZA, DESAI CHENG AND TIN T. TRAN

Abstract. A fundamental inequality for Hilbert spaces is the $\ell_1 - \ell_2$-norm inequality which gives that for any $x \in \mathbb{H}^n$, $\|x\|_1 \leqslant \sqrt{n}\|x\|_2$. But this is a strict inequality for all but vectors with constant modulus for their coefficients. We will give a trivial method to compute, for each x, the constant c for which $\|x\|_1 = c\sqrt{n}\|x\|_2$. Since this inequality is one of the most used results in Hilbert space theory, we believe this will have unlimited applications in the field. We will also show some variations of this result.

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www.ele-math.com
mia@ele-math.com