

## INEQUALITIES FOR CERTAIN POWERS OF POSITIVE DEFINITE MATRICES

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*Abstract.* Let  $A, B$ , and  $X$  be  $n \times n$  matrices such that  $A, B$  are positive definite and  $X$  is Hermitian. If  $a$  and  $b$  are real numbers such that  $0 < a \leq s_n(A)$  and  $0 < b \leq s_n(B)$ , then it is shown, among other inequalities, that

$$\left\| \left\| A^b X + X B^a \right\| \right\| \geq (1 + \min(a^2, b^2)) \|X\|$$

for every unitarily unitarily invariant norm.

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