

JOSIP PEČARIĆ–AND HIS LIFE IN MATHEMATICS AND POLITICS

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Abstract. It is impossible in limited number of pages to give a fair picture of such a remarkable man, great mathematician and human being as Josip Pečarić. Our intention is instead to complement the picture of him in various ways.

We hope that our paper will give also some flavor of Josip as family man, fighter, supervisor, international authority, author (also in other subjects than mathematics), fan of the Croatian football team, and not only as his obvious role as our King of Inequalities.

1. Introduction



Figure 1: Prof. Pečarić in his office

When one speaks on the occasion of someone's 70th birthday and lifetime achievements it should be clear what are the main interests of personal activities.

In the case of Academician Josip Pečarić, supposing you know that he is a great mathematician, you can look at the MathSciNet [1] and find e.g. the following:

- the 1st paper: G. V. Milovanović, J. E. Pečarić, Some considerations on Iyengar's inequality and some related applications, Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. No. 544-576 (1976), 166–170;
- interpolate 1185 items (of which 21 books with the number of citations: $537 > 385 > 343 > 140 \dots$);
- the last paper: S. Ivelić Bradanović, N. Latif, Đ. Pečarić, J. Pečarić, Sherman's and related inequalities with applications in information theory. J. Inequal. Appl. 2018 Paper No.98, 21 pp.

Mathematics subject classification (2010): 01A65.

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Figure 2: MIA2008, Trogir, Croatia

2. The king of inequalities

We and many with us regard him as THE KING OF INEQUALITIES and we are proud to work in his country.

In such a situation it is likely that it will never occur to you to search for any other publicly visible activity after this insight. But for the purpose of thorough CV you can perform a Google search, and soon you realize that related to the name Josip Pečarić appear some books also with non-mathematical titles [3]. You can classify these books as historical, political, maybe even sociological ones.

Since names and people are not in bijective relation it is highly probable that you will disregard this, which many people in Croatia actually also do.

But as a mathematician you continue to verify the hypothesis that these two Josips are actually two different persons. As a final step you maybe search also in Wikipedia where there are two lists of books. One is easily recognizable as a list of mathematical books and the other list (although in Croatian) has no academic titles etc.

Hence your hypothesis broke down and the two Josips collapse into one single person. Moreover, you find that there are indeed 43 non-mathematical books published during the period 1998-2018. REMARKABLE.

So, among other anniversaries, we celebrate also 20 years from publishing his first non-mathematical book.

This is almost beyond comprehension, if you don't know Josip. But if you had a chance to knowing him and, even is better, to work with him, these outcomes assume more human shapes.

His mind never sleeps, always working, searching for problems, quickly switching subjects, searching for solutions, tricky and, afterwards, seemingly simple solutions, or in the words of R. P. Boas Jr. (who was a member of the committee at his PhD dissertation defense 1982):



Figure 3: Josip at a promotion of his book

“Pečarić has systemized enormous amount of material which was published in years and which became more and more chaotic. This systematization itself is a valuable part of the work. Secondly, Pečarić has shown great ingenuity in finding simpler proofs of some inequalities and appropriate generalizations. As a result many inequalities became easily understood. He also unified some previously non-related inequalities. And, finally, he gave many original contributions to the field of inequalities.”

Summing up our initial remarks we again pronounce that it is impossible in a limited space and time to give a fair picture of such a remarkable man, great mathematician and a human being as Josip Pečarić. Our intention is instead to complement the picture of Josip in various ways.

Especially, in addition to his obvious role as king of inequalities, we also hope that this article will give some flavour of Josip as family man, fighter, supervisor, international authority (Australia, Pakistan, Sweden etc.), fan of the Croatian football team, etc.



Figure 4: MIA2008, Trogir, Croatia

Professors: M. Matić, L. E. Persson, V. Stepanov, J. Pečarić and R. Oinarov



Figure 5: Josip with his grandsons

3. The first steps

Josip Pečarić was born in 1948 in town Kotor (Boka Kotorska bay), where he finished elementary and high school.



Figure 6: Boka Kotorska bay

His mother Ivica Tulić was a housewife. His father Ermenedildo Pečarić worked as a postman and later in a restaurant as a waiter. They were a simple and hardworking family with three children. Josip was the youngest child.



Figure 7: Kotor



Figure 8: Josip as a young boy

4. The early career

Josip started studying at the Faculty of Electrical Engineering in Belgrade. One of the professors of mathematics at that faculty was Professor Dragoslav Mitrinović, and most talented mathematicians decided to study at that faculty just because of him.

He also took master's degree in electrical engineering (with supervisor Professor Dobrilo Tošić, which also is a mathematics professor at that faculty). But after that he decided to dedicate himself to pure mathematics.

At the time Gradimir Milovanović (now Academician in Serbia) took his doctoral degree and Professor Tošić gave to Josip a copy of his doctoral thesis, and together with Gradimir Milovanović he wrote his first two articles in 1976. Professor Mitrinović then invited him to work on a doctoral thesis. This was a great privilege (in Josip's own words). Remember that Mitrinović published his famous book "Analytic inequalities" in Springer Verlag in 1970.

Josip finished the work on the PhD thesis in 1979, but he managed to win the degree only in 1982 after a series of attacks on his work. He couldn't get a position as



Figure 9: Josip as a young student, Belgrade 1968



Figure 10: Graduation day

an assistant professor, although he had more articles than his opposing candidates. This was indeed a very hard period for Josip Pečarić. But, as usual, Josip continued to fight for what he judged was correct and we shall all be very grateful for that.

5. The family

The family is very important for Josip. Many times he used to say that his family is a victim of his passions (it is clear by now that these are mathematics and politics). Ankica and Josip founded their family in 1970. They got three children: Ivana, Đilda and Anđela.

It is not known any conference or any kind of scientific travel where Josip was



Figure 11: Prof. D. Mitrinović



Figure 12: PhD graduation ceremony

without his wife Ankica.

6. The continued career in Zagreb

In 1987 Josip moved to University of Zagreb where he was immediately appointed and employed as associate professor, and he became very soon a full professor. The rest is just a remarkable history. Mathematicians are anywhere a wild bunch, so he had to struggle for his place in the Croatian mathematical community but now in entirely different environment. The mathematical school in Croatia was very different from that in Belgrade. Major topics were topology and shape theory, functional analysis, representation theory, geometry and foundations of mathematics (logic). These topics were still under influences of old schools from Austro-Hungarian Empire.



Figure 13: Ankica and Josip



Figure 14: Josip with Ivana, Đilda, Anđela and Ankica

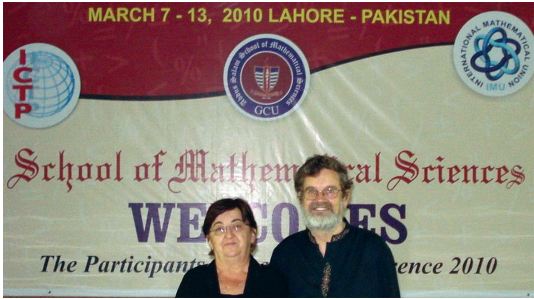


Figure 15: MIA2010, Lahore, Pakistan

It was not easy to convince to contemporary fellow mathematicians that inequalities are worthwhile for researching as an independent subject. Of course, everybody used inequalities, but to establish it as a separate field was a different story. After a while he managed to do that (in some sense). In particular, the around 100 published papers which Josip had in 1987 was something which was very hard to ignore. Very soon he got his first PhD students. Sanja Varošanec was the first who defended her PhD thesis under his supervision in 1994. And many followed. 35 until now. REMARK-ABLE.

7. MIA journal

Having so many published papers with so many international mathematicians it was natural for Josip to think about founding a Croatian international mathematical journal. In Josip's case it is always very short path between an idea and its realization. The first journal he founded (in closed cooperation with Neven Elezović, a publisher and a mathematician, not his student but his most cited coauthor) was:

Mathematical Inequalities & Applications (MIA) which was founded in 1998 with very strong and renowned Editorial Board. Later in 2007 he founded *Journal of Mathematical Inequalities* and *Operators and Matrices*. MIA was very soon cited in World of Science list WoS. Now all three journals are on SCI-Expanded list of mathematical journals.

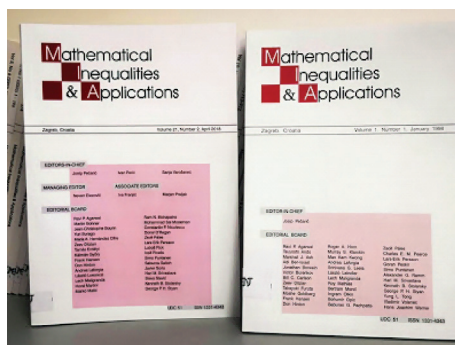


Figure 16: The first and the last number of MIA

Later, Josip published in cooperation with other Croatian and international mathematicians 13 (until now) monographs, under the common name "Monographs in Inequalities", from the field of inequalities with the 1st one in 2005.

8. Josip Pečarić-the academician

Josip was very proud when he was elected as a full member of the Croatian Academy of Sciences and Art (HAZU) in 2000 at the age of 52. He still is very un-



Figure 17: Monographs in inequalities

conventional academician, and in some very conditional and Josip's way "much more militaristic than the norm" (John von Neuman for himself).

There are only four full members of Croatian Academy of Sciences and Art (HAZU) from the field of mathematics. They are: *Goran Muić* (newly elected; representation theory of p -adic groups), *Andrej Dujella* (number theory), *Josip Pečarić* (real functions, theory of inequalities), *Marko Tadić* (representation theory of topological groups). On the next photo you see all these four outstanding mathematicians and behind and over them is the painting of bishop Josip Juraj Strossmayer, the founder of HAZU.



Figure 18: Goran Muić, Andrej Dujella, Josip Pečarić, Marko Tadić

9. The international cooperations

Although numbers of mathematical papers and books produced by Josip are especially remarkable and extremely rare (and anybody can find these numbers on MathSciNet or WoS, see [1, 2]) it should be also emphasized his capability to cooperate with other mathematician and transfer his knowledge and expertise to younger mathematicians. He has, according to MathSciNet, 231 coauthors throughout the world. REMARKABLE.

He has been the direct supervisor to 35 PhD theses and probably at least 20 more would not have been finished without his help. **REMARKABLE.** This is something which cannot be overestimated. What can be of more value than introducing younger people in something so beautiful and inexhaustible as the theory of inequalities.

In particular, Josip has been invited to several countries for longer or shorter research visits. For example, he was invited by the third named author (LEP) and his university at several occasions and this has resulted in 26 joint publications (so far) and one joint PhD student. The early cooperation with Josip was indeed very important for LEP's own career.

There are many similar examples where Josip has substantially influenced the research also in other countries. The maybe most important examples of such visits are Josip's Australia and Pakistan adventures we shortly describe in the next two Sections.

Finally, let us mention that there are examples of "cooperation" we know Josip appreciated very much even if they have not resulted in any joint publication. For example, his friendship with Professor Zvonimir Janko, retired from University of Heidelberg, is very important. He is indeed another great Croatian mathematician who is the eponym of the Janko groups, sporadic simple groups in group theory. Besides summer friendship, they share many things: creativity in mathematics, love to teach younger mathematicians (especially in Croatia, where this is desperately needed), but also political views which are not, say at least, at the bottom of Josip's priorities.



Figure 19: Josip Pečarić and Zvonimir Janko

10. The Australian adventure

Among many scientific travels and conferences probably the most important so far was Josip's visit to Australia for two years in 1992-1993. Among others there he started cooperation with Professors Charles Pierce and Bertram Mond. The work with Mond was especially fruitful. The Mond-Pečarić method was born which resulted in probably more than hundred published papers on operator inequalities in notable international journals.



Figure 20: Bertram Mond, Frank Proschan, Josip Pečarić, 1992

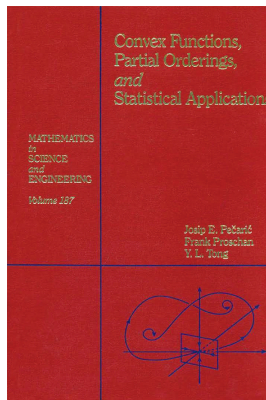


Figure 21: The famous “Red Book”

11. The Pakistan adventure

One international mathematical cooperation which was very dear to Josip was his visiting to Abdus Salam School of Mathematical Sciences (ASSMS), GC University, Lahore, Pakistan. He was invited by Prof. Dr. Alla Ditta Raza Choudary, the Director General of ASSMS. The ASSMS is a unique leading research center of mathematics that has been established in 2003 by the name of famous Pakistani Nobel Laureate of Physics Prof. Dr. Abdus Salam (the founder of ICTP, Trieste). It is a doctoral school in mathematics with no undergraduates. Its faculty is entirely foreign from all over the world (but mostly from Eastern Europe). There is an excellent article about ASSMS in Notices of AMS (Volume 58, Number 7 (938-943)).

It is not easy to track all Josip's visits to ASSMS. According to available data he visited ASSMS from 2006-2014 every year at least for three months. His PhD students

were very motivated and strictly selected. He was the supervisor for 13 PhD theses from 2009 to 2016 (REMARKABLE!) and published many papers with his students.

He left an indelible mark on mathematics in Pakistan. Many of his students now have permanent positions in various universities in Pakistan.



Figure 22: MIA2010, Lahore, Pakistan

12. Concluding words



Figure 23: MIA2018, Zagreb

THANK YOU dear Josip for:

- all wonderful mathematical contributions
- creating the new country of inequalities

- for accepting us as members of this wonderful country
- all you have influenced the mathematical and political areas
- providing us with your fighting spirit of various kinds
- all students you have supervised
- being our dear friend
- and, our cordial congratulation on your 70th anniversary.

Final remark. The basic material in this article is essentially taken from the plenary lecture delivered by the third named author at the MIA2018 Conference in honor of Academician Josip Pečarić on the occasion of his 70th birthday. The basic information for this lecture was collected and selected jointly by all three of us.

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