

## ON HERMITE–HADAMARD TYPE INEQUALITIES FOR HARMONICAL $h$ -CONVEX INTERVAL-VALUED FUNCTIONS

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*Abstract.* We introduce and investigate the concept of harmonical  $h$ -convexity for interval-valued functions. Under this new concept, we prove some new Hermite–Hadamard type inequalities for the interval Riemann integral.

*Mathematics subject classification (2010):* 26D15, 26E25, 28B20.

*Keywords and phrases:* Hermite–Hadamard inequalities, harmonical  $h$ -convexity, interval-valued functions.

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