Mathematical
Inequalities
& Applications
Volume 23, Number 2 (2020), 433–438 doi:10.7153/mia-2020-23-34

A NONEXISTENCE RESULT FOR DISCRETE SYSTEMS RELATED TO THE REVERSED HARDY–LITTLEWOOD–SOBOLEV INEQUALITY

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Abstract. In this paper, we study the Euler-Lagrange system related to the extremal sequences of the discrete reversed Hardy-Littlewood-Sobolev inequality. This system comes into play in estimating bounds of the Coulomb energy and is associated with the study of conformal geometry. By estimating the infinite series, we prove that the nonexistence of super-solutions of the Euler-Lagrange system satisfied by the extremal sequences of this discrete reversed Hardy-Littlewood-Sobolev inequality.

Mathematics subject classification (2010): 26D15, 40B05, 47J20.
Keywords and phrases: Discrete reversed Hardy-Littlewood-Sobolev inequality, Euler-Lagrange equation, nonexistence of positive solution.

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Paper MIA-23-34

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