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WEIGHTED INEQUALITIES OF FEFFERMAN–STEIN TYPE FOR RIESZ–SCHRÖDINGER TRANSFORMS

B. BONGIOANNI, E. HARBOURE AND P. QUIJANO

Abstract. In this work we are concerned with Fefferman-Stein type inequalities. More precisely, given an operator T and some p, $1 < p < \infty$, we look for operators $\mathscr{M}$ such that the inequality

$$\int |Tf|^p w \leqslant C \int |f|^p \mathscr{M} w,$$

holds true for any weight w. Specifically, we are interested in the case of T being any first or second order Riesz transform associated to the Schrödinger operator $L = -\Delta + V$, with V a non-negative function satisfying an appropriate reverse-Hölder condition. For the Riesz-Schrödinger transforms $\nabla L^{-1/2}$ and $\nabla^2 L^{-1}$ we make use of a result due to C. Pérez where this problem is solved for classical Calderón-Zygmund operators.

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REFERENCES

[1] B. BONGIOANNI, E. HARBOURE AND P. QUIJANO, *Weighted inequalities for Schrödinger type singular integrals*, Journal of Fourier Analysis and Applications, 25(3):595–632, 2019.
[2] B. BONGIOANNI, E. HARBOURE AND O. SALINAS, *Riesz transforms related to Schrödinger operators acting on BMO type spaces*, J. Math. Anal. Appl., 357(1):115–131, 2009.
[3] B. BONGIOANNI, E. HARBOURE AND O. SALINAS, *Weighted inequalities for commutators of Schrödinger-Riesz transforms*, J. Math. Anal. Appl., 392(1):6–22, 2012.
[4] A. CORDOBA AND C. FEFFERMAN, *A weighted norm inequality for singular integrals*, Studia Math., 57(1):97–101, 1976.
[5] DAVID V. CRUZ-URIBE, JOSÉ MARIA MARTELL AND CARLOS PÉREZ, *Weights, extrapolation and the theory of Rubio de Francia*, volume 215 of Operator Theory: Advances and Applications, Birkhäuser/Springer Basel AG, Basel, 2011.
[6] J. DZIUBAŃSKI AND J. ZIENKIEWICZ, *Hardy spaces H^1 associated to Schrödinger operators with potential satisfying reverse Hölder inequality*, Revista Matemática Iberoamericana, 15(2):279–296, 1999.
[7] C. FEFFERMAN AND E. M. STEIN, *Some maximal inequalities*, Amer. J. Math., 93:107–115, 1971.
[8] TAO MA, PABLO RAÚL STINGA, JOSÉ L. TORREA AND CHAO ZHANG, *Regularity estimates in Hölder spaces for Schrödinger operators via a $T1$ theorem*, Ann. Mat. Pura Appl. (4), 193(2):561–589, 2014.
[9] C. PÉREZ, *Weighted norm inequalities for singular integral operators*, J. London Math. Soc. (2), 49(2):296–308, 1994.
[10] Z. SHEN, *L^p estimates for Schrödinger operators with certain potentials*, Ann. Inst. Fourier (Grenoble), 45(2):513–546, 1995.
[11] J. MICHAEL WILSON, *Weighted norm inequalities for the continuous square function*, Trans. Amer. Math. Soc., 314(2):661–692, 1989.

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