

## BOUNDEDNESS OF BI-PARAMETER LITTLEWOOD–PALEY $g_\lambda^*$ -FUNCTION ON HARDY SPACES

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*Abstract.* In this paper, we prove the  $H^p - L^p$  boundedness of bi-parameter Littlewood-Paley  $g_\lambda^*$ -function with  $p$  less than 1, improving on the  $H^1 - L^1$  boundedness due to Li and Xue. The main tools include Journé's covering lemma, vector-valued theory and the atomic decomposition of product Hardy spaces.

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