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ON MINMAX AND MAXMIN INEQUALITIES FOR CENTERED CONVEX BODIES

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Abstract. One of the challenging problems from the geometry of (normed or) Minkowki spaces is the question of whether the unit ball must be an ellipsoid if it is a solution of the corresponding isoperimetric problem. The inner and outer radii of the unit ball with respect to the corresponding isoperimetrix (represented in terms of cross-section measures) will be used to establish a result on this problem for a specific measure. Some new minmax and maxmin inequalities for centered convex bodies will also be established.

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