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CONE PROPERTY AND MEASURE DENSITY CONDITION

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Abstract. We prove the existence of open set Ω in the Euclidean space, satisfying the measure density condition, such that the boundary $\partial\Omega$ is a graph and Ω does not satisfy the cone condition. In this way we give an answer to the conjecture formulated by V. Burenkov. Some of the results are formulated in the setting of metric and metric-measure spaces. In particular, for Ω, which is a subset of a metric space, we study the relationships between the measure density condition of Ω and the growth of the measure $\mu(\Omega \cap B(x,r))$, where x is taken from the boundary $\partial\Omega$. Moreover, similar issue is studied for cone condition.

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